2.6/23] $\lim_{x\to 0} \frac{\tan 7x}{\sin 3x} =$

$\lim_{x\to 0} \left(\frac{\tan(7x)}{\sin(3x)}\right)\left(\frac{1}{\sin(3x)}\right)$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\Rightarrow \lim_{x\to 0} \frac{x}{\sin x} = \frac{1}{1} = 1$

$= \lim_{x\to 0} \left(\frac{7}{3}\right)\left(\frac{\sin(7x)}{7x}\right)\left(\frac{1}{\cos(7x)}\right)\left(\frac{3x}{\sin(3x)}\right)$

$= \lim_{x\to 0} \left(\frac{7}{3}\right)\left(1\right)\left(\frac{1}{1}\right)\left(1\right)$

$= \frac{7}{3}$

$$\lim_{x \to 0} 7\frac{\sin(7x)}{7x}$$

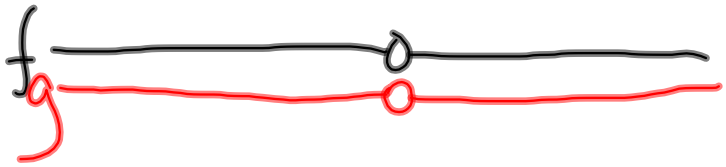
exploit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

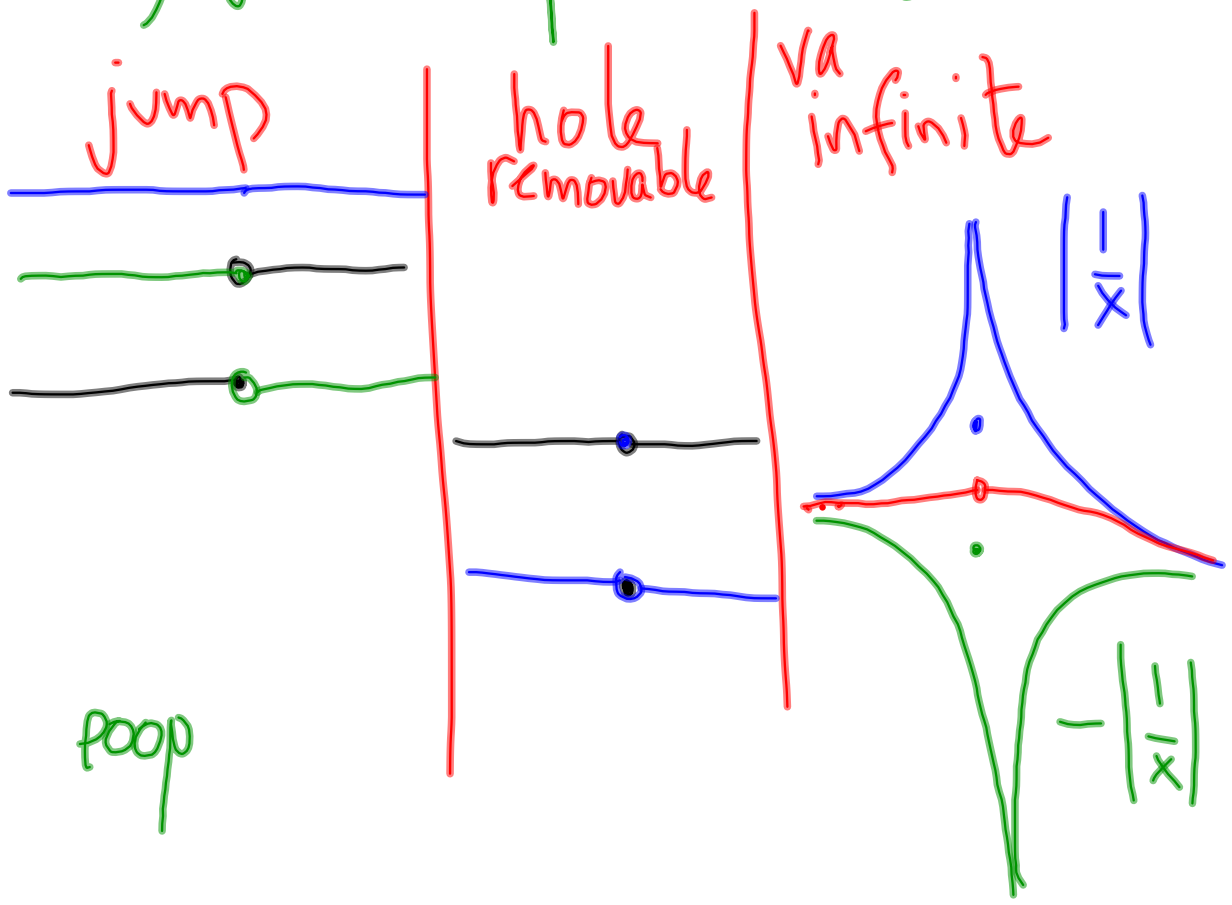
$$= 7 \lim_{x \to 0} \frac{\sin(7x)}{7x} = 7(1)$$

2.5/35) Let f, g be DISCONTINUOUS @ c.

ai) gimme examples where $f+g$ is discontinuous @ c.



aii) gimme 'xamples where $f+g$ is CONT @ c.



3.1) - - - - $f(x)$ continuous

average rate of chg : $\frac{\text{difference in distance}}{\text{difference in time}}$

between $x=a$ and $x=b$

$$= \frac{\Delta f}{\Delta x} \approx \frac{\Delta y}{\Delta x}$$ slope

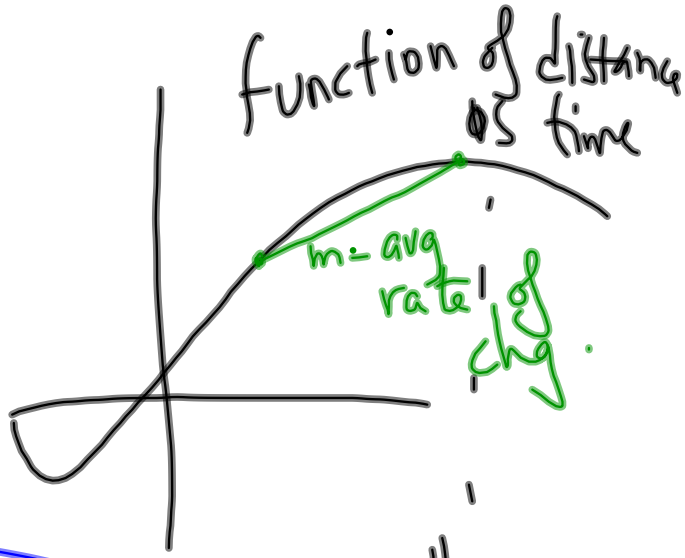
$$= \frac{f(b)-f(a)}{b-a}$$

$$= \frac{f(a)-f(b)}{a-b}$$

$$\frac{\left(\frac{f(b)-f(a)}{b-a}\right)(-1)}{(-1)}$$

$$\frac{-f(b)+f(a)}{-b+a}$$

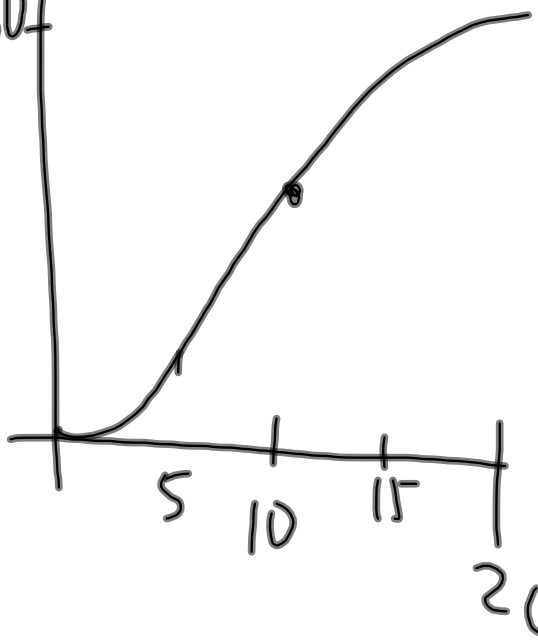
instantaneous rate of chg. of f(x) wrt x



generally fn of y vs x

I can approximate inst. roc w/ avg roc.

3.1/11) 60

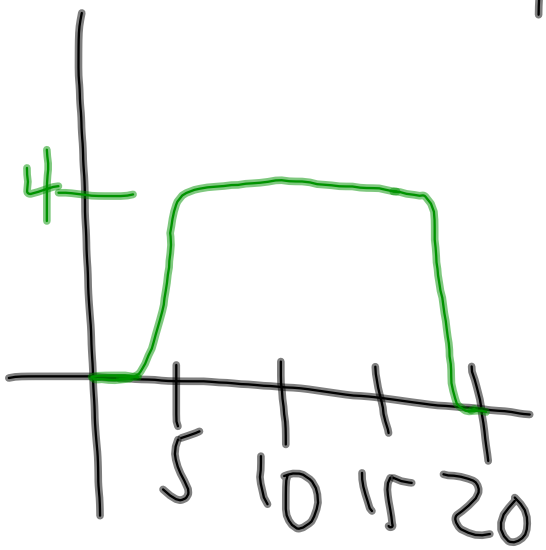


est inst vel @ 10 sec with avg vel between 5 sec & 15 sec.

$$\frac{h(15)-h(5)}{15-5}$$

$$= \frac{50m-10m}{10sec}$$

$$= \frac{40m}{10sec} = 4\ m/sec$$



22/2-6)

$$\lim_{x\to 0}\frac{\sin(6x)}{\sin(8x)}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$=\lim_{x\to 0}\left(\frac{6}{8}\cdot\frac{\sin(6x)}{6x}\right)\left(\frac{8x}{\sin(8x)}\right)$$

$$\lim_{x\to 0}\frac{x}{\sin x}=\frac{1}{1}=1$$

$$=\frac{3}{4}\lim_{x\to 0}\frac{\sin(6x)}{6x}\cdot\lim_{x\to 0}\frac{8x}{\sin(8x)}$$

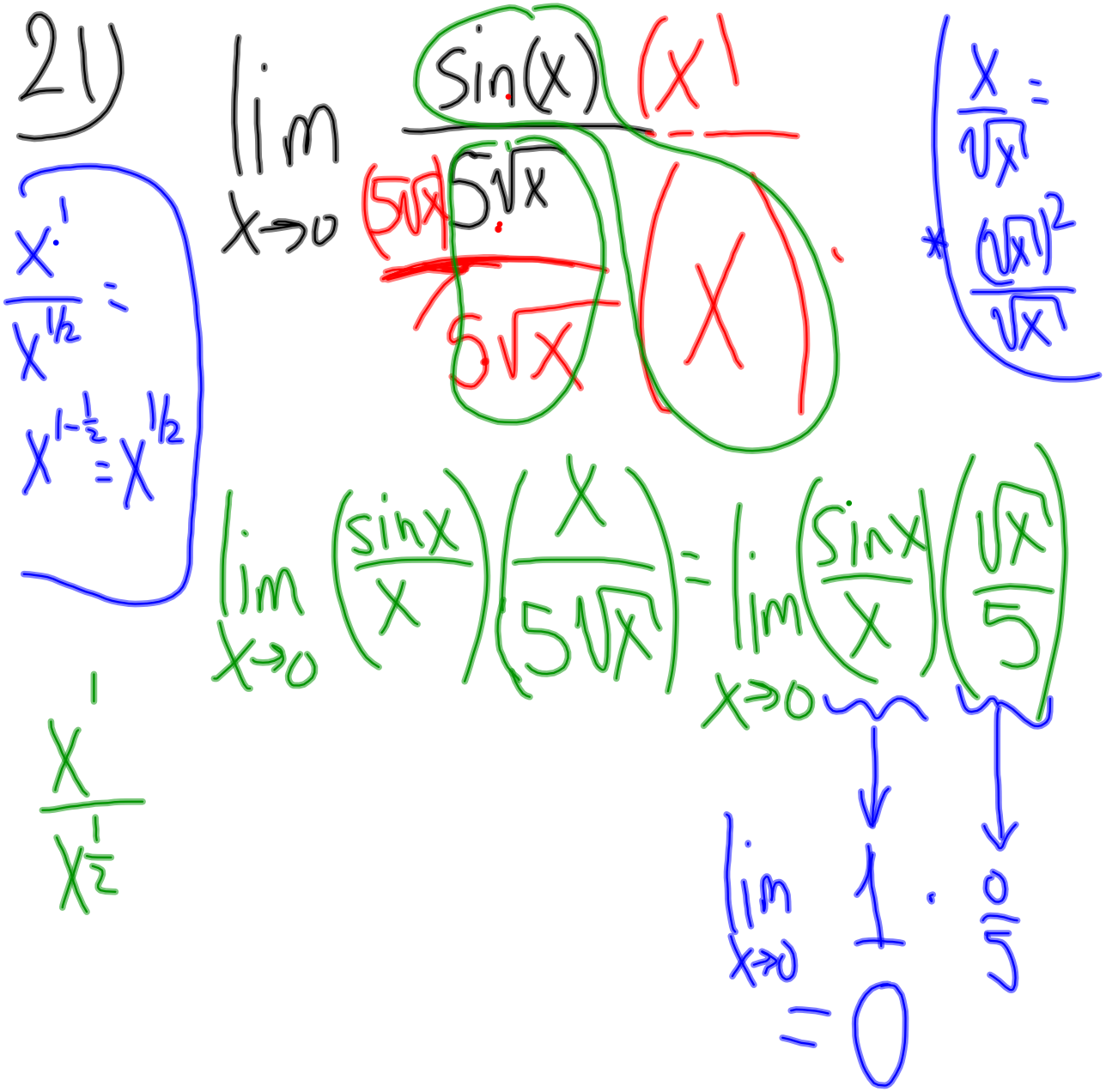
$$\frac{3}{4}(1)(1)=\frac{3}{4}$$

23) $\lim_{x\to 0} \dfrac{\tan(7x)}{\sin(3x)}$

$\lim_{x\to 0} \dfrac{\sin x}{x} = 1$

$$= \lim_{x\to 0}\left(\frac{7}{3}\right)\left(\frac{\sin(7x)}{7x}\right)\left(\frac{1}{\cos(7x)}\right)\left(\frac{3x}{\sin(3x)}\right)$$

$$\left(\frac{7}{3}\right)(1)\left(\frac{1}{\cos(0)}\right)(1) = \frac{7}{3}$$


2)
lim x→0
sin(x)/(5√x)
(x)'
x^1/x^1/2 = x^1-1/2 = x^1/2
X/√X = (√X)^2/√X
lim x→0 (sinx/x)(X/5√X) = lim x→0 (sinx/x)(√X/5)
X^1/X^1/2
lim x→0 1 · 0
= 0

$$\lim_{x \to 0} \frac{\sin x}{5\sqrt{x}} \rightarrow \left(\frac{1}{5\sqrt{x}}\right)\left(\lim_{x \to 0} \frac{\sin x}{1}\right)$$

$$\left(\frac{1}{5\sqrt{x}}\right)\left(\frac{11}{0}\right) = \infty$$

$$\lim_{x \to 0} \frac{1}{5\sqrt{x}} \cdot \lim_{x \to 0} \frac{\sin x}{1}$$

$x \to 0$ ↓ $+\infty$; ↓ 0

INDETERMINATE FORM

24) $\lim_{x \to 0} \frac{\sin^2 x}{x} = \lim_{x \to 0} \left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{1}\right)$

$\downarrow \quad \downarrow$

$1 \cdot 0$

$= 0$

27) $\lim_{x\to 0} \frac{x^2}{1-\cos x}$

$= \lim_{x\to 0} \frac{(x^2)}{(1-\cos x)} \cdot \frac{(1+\cos x)}{(1+\cos x)}$

$= \lim_{x\to 0} \frac{x^2(1+\cos x)}{1-\cos^2 x}$

$= \lim_{x\to 0} \frac{x^2(1+\cos x)}{\sin^2 x}$

$= \lim_{x\to 0} \left(\frac{x}{\sin x}\right)\left(\frac{x}{\sin x}\right)\left(\frac{1+\cos x}{1}\right) = 2$

$1 \cdot 1 \cdot 2$

$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

$\lim_{x\to 0} \frac{x}{1-\cos x} = \frac{1}{0}$

wam!

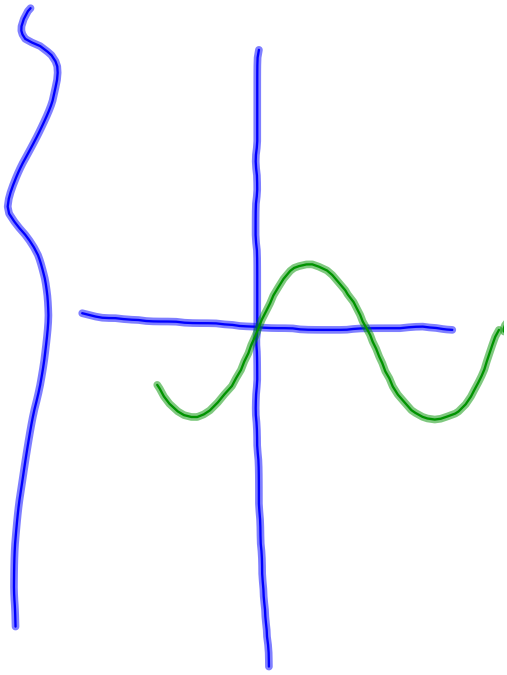
$-\infty$ DNE $+\infty$

$\sin^2+\cos^2=1$

$\sin^2=1-\cos^2$

$\frac{1+\cos 0}{1} = \frac{1+1}{1} = \frac{2}{1} = 2$

28) $\lim_{x \to 0} \frac{x}{\cos\left(\frac{\pi}{2} - x\right)}$



$= \lim_{x \to 0} \frac{x}{\sin(x)} = 1$

x	
0	0
$\frac{\pi}{2}$	1
$\pi/3$	

$\pi/4$, $\pi/6$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$a = \frac{\pi}{2}$, $b = x$: $\frac{\pi}{2} - x$

$\cos\left(\frac{\pi}{2}\right)\cos x + \sin\frac{\pi}{2}\sin x$, with $\sin\frac{\pi}{2} = 1$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$

35) f, g discontinuous at c.

a) $f+g$ is continuous @ c.